Delay-Optimal Cross-Layer Design for Wireless Systems

Vincent Lau
Dept of ECE
Hong Kong University of Science and Technology
May 2009
Outline

- Introduction and Motivation
- Survey of Existing Approaches
- Example I) Delay Optimal SDMA via Stochastic Decomposition
  - Multi-Level Water-Filling Solution
  - Asymptotic Analysis and Numerical Results
- Example II) Delay Optimal OFDMA via Stochastic Learning
  - Convergence Analysis
  - Low Complexity Solution & Asymptotic Optimality
- Conclusion
Introduction and Motivation

Why delay performance is important?

“WHAT??!! He is stuck in the air!! !$*(&#%*!(!”

“You must be kidding me! Buffering at such an important moment!!??”

Fact 1:
Real-life applications are delay-sensitive
Introduction and Motivations

Multiple delay-sensitive applications running at the same time!

Fact II: Different users and applications have **heterogeneous delay requirements**

- Keep track of a game
- Play multi-player game
- Keep talking to some friends
Q) Can’t we just focus on boosting the PHY performance using advanced signal processing techniques (e.g. MIMO)? If the PHY is improved, the delay of the applications will be improved as well. So, why bother to have “cross-layer” design?
Related Works

SDMA Precoder Design for PHY Performance

[Sampath’01], [Scaglione’99],[Palomar’03], etc.
- Dirty Paper Coding (DPC) for MIMO Broadcast Channel
- Zero-Forcing Precoding for SDMA
- assuming knowledge of perfect CSIT.

[Lau’04], [Heath’04], [Love’05] etc.
- Precoder design for SDMA with limited feedback.
- Robust Precoder design for SDMA with outdated CSIT.

Remark:
Only adapt based on CSIT, ignoring queue states and optimize PHY layer performance (throughput) only.

Conclusion: Very important to make use of both (channel state info) CSI and (queue state info) QSI for delay sensitive applications
Introduction and Motivations

Challenges to incorporate QSI and CSI in adaptation

**Challenge 1:** Requires both *Information theory* (modeling of the PHY dynamics) & *Queueing theory* (modeling of the delay/buffer dynamics)

**Challenge 2:** Brute-force approach cannot lead to any viable solution

When Shannon meets Kleinrock...
Various approaches dealing with delay problems

**Approach I : Stability Region and Lyapunov Drift** [Berry’02], [Neely’07], etc.
- Discuss **stability region** of point-to-point SISO and multiuser SISO.
- Also considered asymptotically delay-optimal control policy based on “Lyapunov Drift”
- The authors obtained interesting tradeoff results as well as insight into the structure of the **optimal control policy at large delay regime**.

**Remark:**
This approach allows simple control policy with design insights but the control will be good only for asymptotically **large delay regime**.

Buffer Partitioning

- $S < \frac{1}{v}$
- $S > \frac{1}{v}$

To regulate the buffer state towards $\frac{1}{v}$
Related Works

Various approaches dealing with delay problems

Approach II [Yeh’01PhD], [Yeh’03ISIT]
- Symmetric and homogeneous users in multi-access fading channels
- Using stochastic majorization theory, the authors showed that the longest queue highest possible rate (LQHPR) policy is delay-optimal
Related Works

Various approaches dealing with delay problems

Approach III: [Wu’03], [Hui’07], [Tang’07], etc.
To convert the delay constraint into average rate constraint using tail probability at large delay regime (large derivation theory) and solve the optimization problem using information theoretical formulation based on the rate constraint.

Remark:
While this approach allows potentially simple solution, the control policy will be a function of CSIT only and such control will be good only for large delay regime.

Note:
In general, the delay-optimal power and precoder adaptation should be a function of both the CSI and the QSI.
Related Works

Various approaches dealing with delay problems

Approach IV: [Bertsekas’87]
The problem of finding the optimal control policy (to minimize delay) is cast into a Markov Decision Problem (MDP) or a stochastic control problem.

Remark:
– Unfortunately, it is well-known that there is no easy solution to MDP in general.
– Brute-force value iteration and policy iteration are very complex and time-consuming.
– The curse of dimensionality!!
Related Works

Technical Challenges to be Solved

**Challenge 1:**
A systematic approach for low complexity delay-optimal control policy in general delay regime.

**Challenge 2:**
Exponential Complexity due to coupling among multiple delay-sensitive heterogeneous users.

**Challenge 3:**
Structure of the delay-optimal policy, issue of Limited Buffer Length and Packet Dropping.

**Challenge 4:**
Distributive Implementation??
What shall we do?

Consider two examples to illustrate two techniques for the challenging problem.


Example I) Delay Optimal Power Control in SDMA Systems via Stochastic Decomposition
Introduction

SDMA System

Queueing State Information (QSI)
\[ Q = \{ Q_1, \ldots, Q_L \} \]

Channel State Information (CSI)
\[ H_{N_r \times N_t} \]
System Model

Multiuser MIMO Physical Layer Model

\[ E[XX^H] = I \]

\[ Y_k = h_k \sum_{i=1}^{K} \sqrt{p_i} w_i X_i + Z_k \]
System Model

SDMA Physical Layer Model

Equivalent channel for the $k$-th user:

$$Y_k = \sqrt{p_k} h_k w_k X_k + Z_k$$

Zero-Forcing SDMA:

$$w_k = A_k \left[ I_{Nt} - H_k^T H_k^H \right]^{-1} H_k^T$$

System State ($\chi$): $\chi = \{ h_1, \ldots, h_K \}, \{ Q_1, \ldots, Q_K \}$

Power Allocation Control

$$\mathcal{P} = \{(p_1(\chi), \ldots, p_K(\chi)) : \forall \chi\}$$

Data rate (bits per symbol) of the $k$-th user:

$$R_k(\mathbf{P}) = \log_2 \left( 1 + p_k(\chi) h_k( w_k w_k^H ) h_k^H \right)$$

Power Control

$$R_1(\mathbf{P}) = \log_2 \left( 1 + p_1(\chi) h_1( w_1 w_1^H ) h_1^H \right)$$

$$R_K(\mathbf{P}) = \log_2 \left( 1 + p_K(\chi) h_K( w_K w_K^H ) h_K^H \right)$$
System Model

Queue Dynamics & System States

- G-MAP
  - Packets
  - $Q_1$
  - $\mu_1(Q, H)$

- YouTube
  - Packets
  - $Q_2$
  - $\mu_2(Q, H)$

- MAC State
  - $Q = \{Q_1, Q_2\}$

- SDMA Power Allocation

- MAC Layer
  - SDMA
  - ZF Precoding
  - $H_{N_r \times N_t}$

- Cross Layer SDMA Power Allocation

- PHY Layer
  - PHY Frames
  - Packet Arrivals
  - time

- Channel is quasi-static in a slot
- i.i.d between slots
System Model

**Optimization Objective & Control Policy**

\[ \max_{\mathcal{P}} \sum_{k=1}^{K} Q \cdot \frac{\Pi_k(\mathcal{P})^T \lambda_k}{N_k} \]

S.t.: \[ p_k(\chi) \geq 0 \]

\[ \pi_k(L) \leq \epsilon_d \quad \forall k \in \{1, 2,..., K\} \] (Packet Drop Rate Constraint)

\[ \sum_{k=1}^{K} \mathbb{E}_{\chi}[p_k(\chi)] = \sum_{k=1}^{K} P_k \cdot \Pi_k(\mathcal{P}) \leq P_{avg} \] (Average Power Constraint)

**Challenges:**

- Huge dimension of variables involved (policy = set of actions over all system state realizations)
- K queues are coupled together \( \rightarrow \) Exponentially Large State Space
- Problem not convex
System State Evolution

- **Embedded Markov Chain**

  Sample the continuous time random process $\chi(t)$ at frame boundaries $\{0, \tau, 2\tau, \ldots\}$, we have an “embedded discrete time random process”:

  $\chi_m = (H_m, Q_m)$ where $\chi_m = \chi(m\tau)$

**Lemma 1)** For a given control policy, the embedded random process $\chi_m = (H_m, Q_m)$ is a Controlled Markov chain with transition kernel given by:

$$
\Pr[H_{m+1}, Q_{m+1}|\chi_m, p(\chi_m)] = \prod_{k=1}^K \Pr(h_{k,m+1}) \Pr[Q_{m+1}|\chi_m, p(\chi_m)]
$$
System State Evolution

Sketch of Proof

Given the current state \( \chi_m = (H_m, Q_m) \) and the control action \( p_k(\chi_m) \), one of the following events could occur for user \( k \) at the \((m+1)\)-th scheduling slot.

Packet arrival from the data source: Since packet arrival follows Poisson distribution with mean arrival rate \( \lambda_k \), the transition probability of the buffer state corresponding to packet arrival is given by:

\[
p_{k,q,q+1} = \Pr[Q_{k,m+1} = q + 1 | Q_{k,m} = q] = 1 - e^{-\lambda_k T} \quad \text{for } q < L
\]  

Packet drop due to limited buffer size:

\[
\eta_k = \frac{\Pr(\text{Packet arrival} | Q_{k,m} = L) \Pr[Q_{k,m} = L]}{\Pr(\text{Packet arrival})} = \frac{\lambda_k T \Pr[Q_{k,m} = L]}{\lambda_k T} = \Pr[Q_{k,m} = L]
\]  

Since the inter-arrival time of packets is memoryless, the above probabilities in (4) and (5) (conditioned on \( \chi_m \)) is independent of the previous system states \( \{\chi_{m-1}, \chi_{m-2}, \ldots\} \).
System State Evolution

Sketch of Proof

Packet departure from the data buffer: A packet can depart if and only if the required service time of the remaining packet is no more than one slot duration. Since the packet length is exponentially distributed with mean packet length $\bar{N}_k$, the probability for packet departure at $t = (m+1)\tau$ (conditioned on the system state $\chi_m$) is given by:

$$
\begin{align*}
    p_{k,q,q-1} &= \Pr[Q_{k,m+1} = q-1|Q_{k,m} = q, \chi_m, p_k(\chi_m)] \\
    &= \Pr\left(\frac{1}{\log_2(1+p_k(\chi))} \mu_k(\chi) = \frac{\log_2(1 + p_k(\chi)h_k w_k^H h_k^H)}{\bar{N}_k} \right) \\
    &= \Pr\left(\frac{N_k}{\bar{N}_k} < \mu_k(\chi_m)\tau\right) = 1 - e^{-p_k(\chi_m)\tau} \approx p_k(\chi_m)\tau
\end{align*}
$$

(6)

Since the packet length $N_k$ is memoryless, the above probability (6) (conditioned on $\chi_m$ and action $p_k(\chi_m)$) is independent of the system state $\{\chi_{m-1}, \chi_{m-2}, \ldots\}$.

As a result of the memoryless property of the packet interarrival and packet length distribution as well as (3), the embedded random process $\chi_m = (Q_m, H_m)$ is a discrete time Markov process. Furthermore, since $\lambda_k\tau$ and $\mu_k\tau$ are small, the probability of multiple packet arrivals or packet departures is of the order $O[(\lambda_k\tau)^2]$ and hence is negligible.
Our Transition Probability Kernel:

State transition diagram for K-dimension Markov chain \( \{Q_m\} \) with \( N \) states each dimension. \( K=2 \) for illustration.

For unichain control policy, the induced Markov Chain is “aperiodic” and “irreducible”.

Power Control in PHY \( \Rightarrow \) Controlled Service Rate in Queues
Technical Challenges

Major Challenges

1) Exponentially large Q state (QSI):
   - The total number of states in the joint-queue-state (QSI) = $N^L$
   - Exponentially large $\rightarrow$ complexity and memory requirement = $O(\exp[L])$

2) Global Optimal Solution:
   - The problem is not convex. How to make sure we have global optimal solution?

3) Asymptotic Analysis:
   - Any useful insights can be obtained on the structure of delay-optimal solution? How to do buffer dimensioning?
Problem Decomposition

Primal Decomposition

Define auxiliary variables:

$$P_k = P_k \cdot \Pi_k(P);$$
$$P_{main} = \{P_1, P_2, ..., P_K\}$$

The optimization problem becomes:

$$T^* = \min_{P_{main}} \sum_{k=1}^{K} \frac{U_k \tau}{\lambda_k}$$

$$U_k = Q \cdot \Pi_k(P)$$

S.t:  $$p_k(x) \geq 0$$

$$\pi_k(L) \leq \epsilon_d \ \forall k \in \{1, 2, ..., K\}$$

$$\sum_{k=1}^{K} P_k \leq P_{avg}$$
Problem Decomposition

Primal Decomposition

For given $P_{\text{main}}$, $U_k$ is a function of $P_k$ only and hence, we have:

$$\min_{P_{\text{main}}, P_k} \sum_{k=1}^{K} \frac{U_k \tau}{\lambda_k} = \min_{P_{\text{main}}} \sum_{k=1}^{K} \min_{P_k} \frac{U_k \tau}{\lambda_k}$$

As a result, we can decompose the problem into one master problem + K subproblems

Problem 1 (Master Problem):

$$\overline{T}^* = \min_{P_{\text{main}}} \sum_{k=1}^{K} \frac{U_k^*(P_k) \tau}{\lambda_k}$$  \hspace{1cm} (18)

S.t.: $$\sum_{k=1}^{K} P_k \leq P_{\text{avg}}$$  \hspace{1cm} (19)

Average Power allocation to the K users
Problem Decomposition

Primal Decomposition

Problem 2 (Sub Problem):

\[ \bar{U}_k^*(P_k) = \min_{P_k} Q \cdot \Pi_k(P) \]  \hspace{1cm} (20)

S.t.: \[ p_k(\chi_k) \geq 0 \]  \hspace{1cm} (21)

\[ \tau_k(L) \leq \epsilon_d \]  \hspace{1cm} (22)

\[ P_k \cdot \Pi_k(P) = \bar{P}_k \]  \hspace{1cm} (23)

Note that given the power allocation matrices \( P_k \), the users evolve according to their own local dynamic equations. Hence, we could write \( P_k \) as the evolution law for user 1 and QSI only.

Instantaneous power allocation to the k-th user (subject to k-th user average power constraint \( \bar{P}_k \)).
Solution of the Subproblem

Transformation of Variables

- The subproblem is not convex w.r.t. the optimization variables \( \{p_k(\chi_k)\} \)

- Using birth death dynamics of the problem, the subproblem is equivalent to:

\[
U_k^* = \min_{P_{k,q}} \frac{\sum_{q=0}^{L} \frac{\prod_{l=q+1}^{L} \mu_{k,l}(P_{k,l})}{\lambda_k^N-q}}{\sum_{q=0}^{L} \frac{\prod_{l=q+1}^{L} \mu_{k,l}(P_{k,l})}{\lambda_k^N-q}}
\]

S.t.: \[
\frac{1}{\sum_{q=0}^{L} \frac{\prod_{l=q+1}^{L} \mu_{k,l}(P_{k,l})}{\lambda_k^N-q}} \leq \epsilon_d
\]

\[
P \cdot \Pi(P_k) = \frac{\sum_{q=0}^{L} \frac{\prod_{l=q+1}^{L} \mu_{k,l}(P_{k,l})}{\lambda_k^N-q} P_{k,q}}{\sum_{q=0}^{L} \frac{\prod_{l=q+1}^{L} \mu_{k,l}(P_{k,l})}{\lambda_k^N-q}} \leq P_k
\]

\[
\bar{\mu}_{k,q}(P_{k,q}) = \max_{\bar{P}_{k,q}} \mathbb{E}_H[\mu_{k,q}(\chi)|Q_{k,m} = q]
\]

\[
\bar{P}_{k,q} = \mathbb{E}[p_k(\chi_x)|Q_k = q]
\]
Solution of the Subproblem

Transformation of Variables

Consider the following transformation: \( v_{k,q} = \prod_{i=q+1}^{L} \frac{\bar{u}_{k,i}}{\lambda_k}, \quad q \in \{0, 1, \ldots, L\} \)

(One-to-one mapping)

\( \mathcal{V}_k = \{\mu_k, 0, \ldots, \mu_k, L\} \leftrightarrow \mathcal{P}_k = \{\bar{P}_k, 0, \ldots, \bar{P}_k, L\} \)

Transforming from the P domain to the V domain, the subproblem is equivalent to:

\[
\begin{align*}
\overline{U}_k^* &= \min_{\mathcal{V}_k} \frac{\sum_{q=1}^{L} qv_{k,q}}{\sum_{q=0}^{L} v_{k,q}} \\
\text{S.t.} \quad \frac{1}{\sum_{q=0}^{L} v_{k,q}} &\leq \epsilon_d \\
\sum_{q=1}^{L} F\left(\frac{v_{k,q-1}}{v_k, q}\right) v_{k,q} &\leq \bar{P}_k
\end{align*}
\]

\[
\begin{align*}
\min_{\overline{U}_k, \mathcal{V}_k} \overline{U}_k \\
\text{S.t.} \quad \sum_{q=1}^{L} qv_{k,q} - \overline{U}_k \sum_{q=0}^{L} v_{k,q} &\leq 0 \\
1 - \epsilon_d \sum_{q=0}^{L} v_{k,q} &\leq 0, \quad \sum_{q=1}^{L} F\left(\frac{v_{k,q-1}}{v_k, q}\right) v_{k,q} - \bar{P}_k \sum_{q=0}^{L} v_{k,q} &\leq 0
\end{align*}
\]
Solution of the Subproblem

Global Optimal Solution

Theorem (Unique optimal solution): The subproblem has a unique global optimal solution. Furthermore, the following algorithm can reach the solution in $\lceil \log_2 \left( \frac{L}{\varepsilon} \right) \rceil$ steps.

Algorithm 1 (Bisection Searching):

- **Initialize**: Set $\overline{U}_{\text{min}} = 0; \overline{U}_{\text{max}} = L$.
- **Repeat**:
  - Set $\overline{U}_k = \frac{\overline{U}_{\text{min}} + \overline{U}_{\text{max}}}{2}$
  - Solve Problem 4 (defined below) using Algorithm 2;
  - **if** the optimal solution of Problem 4 $S_{\min} \leq 0$, $\overline{U}_{\text{min}} = \overline{U}_k$, **else** $\overline{U}_{\text{max}} = \overline{U}_k$;
- **Until** $\overline{U}_{\text{max}} - \overline{U}_{\text{min}} < \varepsilon$ where $\varepsilon$ is the performance error tolerance bound. $\overline{U}_k^* = \overline{U}_k$. 
Solution of the Subproblem

Structure of the Optimal Solution

- **Multi-level Power Water-filling:**
  
  Water-level adaptive to QSI

\[
p_k^*(x_k) = \left[\frac{1}{\alpha_{k,q}^* N_k} \left(1 - \frac{1}{h_k(w_k w_k^H h_k^H)}\right)\right]^+
\]

- The water levels \(\{\alpha_{k,q}^*\}\) can be determined offline based on long-term statistical information of the data source and CSI.

- Memory requirement is \(O(L)\)

Power Allocation according to water-filling w.r.t. CSI of users
Solution of the Master Problem

- Recall that the master problem is to determine the “average power allocation” to the SDMA users \( P_{\text{main}} = \{ \overline{P}_1, \ldots, \overline{P}_K \} \)

- \( U_k^*(\overline{P}_k) \) is a convex function of \( \overline{P}_k \) \( \Rightarrow \) The master problem is convex in \( \{ \overline{P}_1, \ldots, \overline{P}_K \} \)

- Form the Lagrangian function for the master problem

---

**Lemma 4.3 (Derivative of optimal buffer length w.r.t power constraint):** Denote the lagrange multipliers corresponding to the optimal scheme \( \mathcal{V}_k = \{ v_{k,q}^* \} \) of Problem 4 when \( \overline{U}_k = \overline{U}_k^* \) as \( \beta_{k1}^*, \beta_{k2}^* \). The derivative of \( U_k^*(\overline{P}_k) \) w.r.t. \( \overline{P}_k \) in the sub problem is given by (42). Moreover, \( \frac{\partial U_k^*}{\partial \overline{P}_k} \) is a non-decreasing function of \( \overline{P}_k \).

\[
\frac{\partial U_k^*}{\partial \overline{P}_k} = -\frac{\beta_{k2}^*}{1 - \beta_{k1}^* - \beta_{k2}^*}
\] (42)

---

- How to determine the subgradient?
We consider high SNR scenario

Lemma 5.2 (Asymptotic closed-form expression of \( \{\alpha_{k,q}^*\} \) in terms of \( \alpha_{k,1}^* \)): The water-filling levels under different QSIIs is an geometric series:

\[
\frac{1}{\alpha_{k,q} N_k} = O \left( \left( \frac{\log \left( \frac{1}{\alpha_{k,1}^* N_k} \right) }{\frac{1}{\lambda_k N_k}} \right)^{q-1} \frac{1}{\alpha_{k,1} N_k} \right), \quad q \in \{1, 2, ..., L\}. \tag{45}
\]

**Fig. 3.** Relationship of water levels in the proposed multi-level water-filling solution. The y-axis is log of water level and the x-axis is the QSI. We assume \( L = 10 \) and \( SNR = 10 \log_{10}(P_k) \)

\( \log \left( \frac{1}{\alpha_{k,q}^*} \right) \) forms an arithmetic series

\[\rightarrow \{\alpha_{k,q}^*\} \text{ forms a geometric series}\]
Asymptotic Analysis

Corollary 5.1 (Performance gain compared to the CSI-only policy): Optimal buffer length $\overline{U}_k^*$ achieved by the proposed multilevel water-filling algorithm is

$$\frac{\lambda_k}{\mathcal{O}(\log P_k) + \mathcal{O}(\log \log P_k) - \lambda_k}$$

while that achieved by the traditional CSI-only (single-level water-filling) policy is

$$\mathcal{O}(\log P_k) - \lambda_k.$$

Gain due to multi-level water-filling

Substantial delay gain vs CSIT-only scheme
Asymptotic Analysis

Buffer Length Requirement

**Corollary 5.2 (Minimum power required due to finite buffer size):** Denote $P_{k,\text{min}}$ as the minimum power to achieve the packet drop rate constraint $\varepsilon_d$ under a maximum buffer size $L$.

$$\log \log (P_{k,\text{min}}) = \log \left( \frac{-\log \varepsilon_d}{L} \right) + \log(\lambda_k) + \log(N)$$

(48)

First order guideline on buffer dimensioning

For small $\varepsilon_d$

$$[\log \log SNR_{min}] \times L = \text{constant}$$
Conclusion

**Conclusion 1 (Structure of Delay-Optimal Power Control):**
Delay-Optimal Power Allocation – **multilevel water-filling**: Water-filling across CSI, water level determined by QSI.

**Conclusion 2 (Complexity):**
Low complexity \( O(K) \) solution via stochastic decomposition and birth-death queue dynamics

**Conclusion 3 (Asymptotic Results):**
Gain of multilevel water-filling is \( \log \log \) SNR.

**Buffer Length \( \times \log \log \) SNR = constant**
Example II) Delay Optimal Power and Subband Allocation in OFDMA Systems via Stochastic Learning
OFDMA System Model

CSI from the K mobiles

QSI of the K queues

Resource Allocation Controller

SC alloc. policy $\Omega_S$
P alloc. policy $\Omega_P$

$\lambda_1$  $\lambda_2$  $\lambda_3$  

$\mu_1(Q)$  $\mu_2(Q)$  $\mu_K(Q)$

$Q_k$  $Q_2$  $Q_1$
OFDMA PHY Model

OFDMA Physical Layer Model

\[ \lambda_1, \mu_1(Q, H) \]

\[ \lambda_2, \mu_2(Q) \]

\[ Q_2 \]

\[ \lambda_L, \mu_L(Q, H) \]

\[ Q_L \]

\[ \{X_1\} \]

\[ \{X_{k,n}\} \]

Subband & Power Allocation

OFDMA PHY

Data Rate \( R_k \)

\[ R_k = \sum_{n=1}^{N_F} s_{k,n} I(X_{k,n}; Y_{k,n}|H_{k,n}) = \sum_{n=1}^{N_F} s_{k,n} \log(1 + p_{k,n}|H_{k,n}|^2) \]

\[ Y_{k,n} = H_{k,n}X_{k,n} + Z_{k,n} \]

\[ \mathbb{E}[XX^H] = I \]
OFDMA Queue Dynamics

- Time domain partitioned into scheduling slots
- CSI $H(t)$ remains quasi-static within a slot and iid between slots
- Packet arrival $A(t) = (A_1(t), ..., A_K(t))$ where $A_k(t) \sim$ iid according to a general distribution $P_k(A)$.
- $N_k(t)$ denotes the random packet size $\sim$ iid.
- $Q(t)$ denotes the number of packets waiting in the buffer at the $t$-th slot.

$$Q_k(t+1) = \min\{[Q_k(t) - R_k(t) \tau / N_k(t)]^+ + A_k(t), N_Q\}$$

- Global System State (CSI, QSI)$\chi(t) = (H(t), Q(t))$

Total number of bits Transmitted in the $t$-th slot
**OFDMA Delay-Optimal Formulation**

**Stationary Power and Subband Allocation Control Policy**

- A mapping \( \Omega = (\Omega_p, \Omega_s) \) from the system state \( \chi \) to a power and subband allocation actions.

\[
\Omega_p(\chi) = \{p_{k,n}\} \quad \Omega_s(\chi) = \{s_{k,n}\}
\]

- Power Constraint:

\[
\sum_{k=1}^{K} \sum_{n=1}^{N_F} \mathbb{E}[p_{k,n}] \leq P_0, \quad p_{k,n} \geq 0,
\]

- Subband Allocation Constraint:

\[
\sum_{k=1}^{K} s_{k,n} = 1 \quad \forall n \in \{1, N_F\}
\]
OFDMA Delay-Optimal Formulation

Problem Formulation

Find the optimal control policy $\Omega$ that minimizes

$$J^\Omega_{\beta} = \sum_{k=1}^{K} \beta_k T_k(\Omega) + \gamma P_{tx}(\Omega) = \lim_{T \to \infty} \frac{1}{T} \sum_{t=1}^{T} \mathbb{E}[g(\chi(t), \Omega(\chi(t)))]$$

“Positive Weighting Factor”

$\beta = (\beta_1, \beta_2, \cdots, \beta_L)$

Pareto Optimal delay boundary

$P_{tx}(\Omega) = \lim_{T \to \infty} \frac{1}{T} \sum_{t=1}^{T} \mathbb{E} \left[ \sum_{n=1}^{N} p_{k,n}(t) \right] = \mathbb{E}_{\pi_x} \left[ \sum_{k,n} p_{k,n} \right] \leq P_0$

“Per-stage reward”

$$g(\chi, \{p, s\}) = \sum_{k} \beta_k Q_k + \gamma \sum_{k,n} p_{k,n}$$

$[Q_k] \forall k \in \{1, K\}$
Infinite Horizon Average Reward MDP

Given a stationary control policy \( \Omega \), the random process \( \{\chi(t), g(\chi(t), \Omega(\chi(t)))\} \) evolves like a Markov Chain with transition kernel:

\[
\Pr[\chi(t+1)|\chi(t), \Omega(\chi(t))] = \Pr[H(t+1)]\Pr[Q(t+1)|\chi(t), \Omega(\chi(t))]
\]

Solution is given by the “Bellman Equation”

\[
\theta + V(\chi^i) = \min_{u(\chi^i)} \left[ g(\chi^i, u(\chi^i)) + \sum_{\chi^j} \Pr[\chi^j|\chi^i, u(\chi^i)]V(\chi^j) \right]
\]

“Potential function” (contribution of the state \( i \) to the average reward)

“Optimal Value” \( \theta = J^*_\beta = \inf_\Omega J^*_\beta \)

\((N_Q + 1)^K\) Equations and \((N_Q + 1)^K + 1\) unknowns
Optimal Solution

Example of the Solution Structure

For the special case of exponential packet length $N(t)$ and Poisson Arrival, the optimal power and subband control are given by:

$$p_{k,n}(H, Q^i) = s_{k,n}(H, Q^i) \left( \frac{\Delta \tilde{V}(Q^i_k)}{N_k} - \frac{1}{|H_{k,n}|^2} \right)^+$$

$$s_{k,n}(H, Q^i) = \begin{cases} 1, & \text{if } X_{k,n} = \max_k \{X_{k,n}\} > 0 \\ 0, & \text{otherwise} \end{cases}$$

Water-level depends on QSI (via potential function)

$$\Delta \tilde{V}(Q^i_k) = \tilde{V}(Q^i_1, \ldots, Q^i_K) - \tilde{V}(Q^i_1, \ldots, [Q^i_k - 1]^+, \ldots, Q^i_K)$$

Subband Allocation Metric (depends on both CSI and QSI)

$$X_{k,n} = \frac{\Delta \tilde{V}(Q^i_k)}{N_k} \log \left( 1 + |H_{k,n}|^2 \left( \frac{\Delta \tilde{V}(Q^i_k)}{N_k} - \frac{1}{|H_{k,n}|^2} \right)^+ \right) - \gamma \left( \frac{\Delta \tilde{V}(Q^i_k)}{N_k} - \frac{1}{|H_{k,n}|^2} \right)^+$$
Optimal Solution

How to determine the potential function?

Brute-Force solution of the Bellman Equation?:
- Too complicated, exponential complexity and memory requirement

Online stochastic learning?
- Iteratively estimate potential function based on observation – online value iteration
- Due to exponentially large state space, convergence speed is an issue (not scalable w.r.t. K)

How to break this “scalability barrier”?
Optimal Solution

Definition 3: [Semi-Global Subcarrier Allocation Policy] A semi-global subcarrier allocation policy is defined as $\tilde{\Omega}_a(\mathbf{H}, Q) = \{\tilde{s}_{k,n}(\mathbf{H}, Q_k) \in \{0, 1\} | \sum_{k=1}^{K} \tilde{s}_{k,n} = 1 \forall n\}$. In other words, the subcarrier allocation $\tilde{s}_{k,n}(\mathbf{H}, Q_k)$ of the $k$th user in the $n$th subcarrier is a function of the global CSI $\mathbf{H}$ and the local QSI $Q_k$ only.


$\tilde{V}(Q) = \sum_k \tilde{V}_k(Q_k)$

$(\theta_k, \{\tilde{V}_k(Q_k)\})$ is the solution of the “per-user Bellman equation”

$\theta_k = \min_{u_k(Q_k)} \tilde{g}_k(Q_k, u_k(Q_k)) + \lambda_k \tau \Delta \tilde{V}_k(Q_k + 1) - \overline{u}_k(Q_k) \tau \Delta \tilde{V}_k(Q_k),$

Complexity $\sim O(K) \Rightarrow$ Much faster convergence when applying online Stochastic learning on the “per-user Bellman equation” (Convergence proof skipped)

Corollary: The semi-global subband allocation policy is asymptotically optimal for large $K$. 
Numerical Results

Average Delay per user vs SNR

The number of users $K = 2$, the mean packet size $N_k = 1526$ Kbyte/pt,

Huge gain in delay performance Compared with conventional CSIT only schemes and RR

Close-to-optimal performance even for small # of users
Numerical Results

Average Delay per user vs number of iterations
Number of users $K=16$, average packet size = 10kbyte
Transmit SNR = 10dB

Fast convergence ("lock-in") of the online stochastic learning algorithm
Conclusion

**Conclusion 1 (Structure of Delay-Optimal Power Control and Subband Allocation):**
- Power Allocation – multilevel water-filling: Water-filling across CSI, water level determined by QSI.
- Subband Allocation – choose the user with the largest metric \( f(QSI, CSI) \)

**Conclusion 2 (Complexity):**
Under “semi-global subband allocation”, we derive a low complexity \( O(K) \) solution via stochastic learning

**Conclusion 3 (Asymptotic Results):**
Semi-global subband allocation is “asymptotically optimal” for large \( K \).
References


Thank you!

Questions are Welcomed!

Vincent Lau - eeknlau@ee.ust.hk

http://www.ee.ust.hk/~eeknlau