# PERFORMANCE ANALYSIS AND COMPARISONS BETWEEN MLED AND OFDM IN SPACE-TIME TRELLIS CODING OVER MIMO FREQUENCY SELECTIVE FADING CHANNELS†

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Daniel Ka Chun So, *Student Member, IEEE* and Roger S. Cheng, *Member, IEEE*
Department of Electrical and Electronic Engineering
The Hong Kong University of Science and Technology
Clear Water Bay, Hong Kong
Email: danielso@ee.ust.hk, eecheng@ee.ust.hk

## ABSTRACT

Space-time trellis code (STTC) in frequency selective fading channel using maximum likelihood equalization and detection (MLED), and orthogonal frequency division multiplexing (OFDM) are compared in this paper for channels with arbitrary delay profile. The performance bound for both schemes are first derived and their performances are compared both analytically and through simulations. Code design, receiver complexity, interleaver design and the robustness issues are addressed in these comparisons.

*Index Terms* - Intersymbol interference, maximum likelihood detection, OFDM modulation, performance evaluation, space-time coding

## I. INTRODUCTION

Space-time trellis code (STTC) [1] is one of the promising techniques for multi-input multi-output (MIMO) wireless communication systems. Providing higher capacity than single antenna systems, STTC also has good performance by capitalizing on the spatial and temporal diversity. For an ($N$, $M$) space-time trellis coded system with $N$ transmit and $M$ receive antennas, its maximum achievable diversity order is $N\times M$ over flat fading channels. As many wireless channels are frequency selective, research works have already been done on applying STTC over these channels [2,3,4,5,6]. There are mainly two categories of transmission and detection schemes: 1) the maximum likelihood equalization and detection (MLED), and 2) the orthogonal frequency division multiplexing (OFDM).

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The MLED scheme uses trellis equalization and detection based on maximum likelihood rule. For STTC, the first MLED scheme appeared in [2], where an iterative (Turbo) maximum aposteriori equalizer and decoder (MAP-ED) was proposed for systems with and without interleaving. However, it has extremely high complexity that prohibits its practical usage. A reduced complexity approach using combined trellis equalizer and decoder (CT-ED) was later proposed in [3]. However, its complexity still grows exponentially with the channel tap length. Various schemes on reducing the complexity of STTC MLED were proposed, such as the IQ decoupled Turbo equalizer [4], and the reduced state-space method with decision feedback for non-interleaved systems [3].

Another approach for applying STTC in frequency selective fading channels is to use OFDM. It combats frequency selectivity by transforming the frequency selective fading channel into a number of parallel flat fading subchannels. The OFDM modulated STTC system was first presented in [5]. The performance analysis and code design of STTC OFDM for uniform profile channel was discussed in [6], and the achievable diversity order stated is $N \times M \times L$, where $L$ is the number of multipaths.

These two different schemes are capable of handling frequency selectivity for STTC. However, to our knowledge, there is no analytical comparison between these two schemes and their comparative advantages over various environments are also unknown. In this paper, we aim at comparing the performances and code design issues between these two major schemes both analytically and empirically. The pairwise error probability bound for STTC MLED is derived and generalized for STTC OFDM from uniform to arbitrary channel delay profile. The maximum diversity order achievable by these schemes are compared. The issues of code design, receiver complexity, interleaver design and robustness are also discussed.

This paper is organised as follows. In Section II, the STTC system with MLED is presented, and the STTC OFDM approach is briefly revisited in Section III. Section IV of this paper discusses the code design issues for these schemes, and Section V provides a throughout comparison between them through analysis and simulations. Some conclusions are drawn at Section VI.

## II. STTC MLED

*A. Signal Model*

Consider an ($N$, $M$) STTC system with $N$ transmit and $M$ receive antennas. The data are encoded and mapped into $N$ streams of modulated symbols by the space-time encoder, which are then transmitted simultaneously using its respective transmit antennas. The system is transmitted through a quasi-static frequency selective fading channel, with all $N \times M$ channels possessing independent fades. These channels are modelled as symbol-spaced tap delay lines with $L$ taps. In wireless wave propagation, the delay profiles of the channels are mainly affected by reflection off large objects. Therefore, as the antennas are not separated very far apart, all $N\times M$ channels should have similar delay spread. Hence, the tap lengths for all these channels are assumed to be equal. The channels are further assumed to be slowly time varying with quasi-static fading, and have an arbitrary delay profile.

Let $c_i(t)$ be the coded symbol transmitted in transmit antenna $i$ at time $t$ (with a frame length of $T$), and the coded symbol vector be $\mathbf{C}_i(t)=[c_i(t),c_i(t-1),\ldots,c_i(t-L+1)]$. Also, let $h_{i,j}(l)$ be the $l$-th channel tap gain from transmit antenna $i$ to receive antenna $j$, and the symbol spaced channel vector be $\mathbf{h}_{i,j}=[h_{i,j}(0),h_{i,j}(1),\ldots,h_{i,j}(L-1)]$. The additive white Gaussian noise received by receive antenna $j$ at time $t$ is denoted as $n_j(t)$, with variance $N_0/2$ per dimension. Thus the received signal in antenna $j$ at time $t$ is $r_j(t)=\mathbf{H}_j\mathbf{C}(t)+n_j(t)$ with $\mathbf{H}_j=[\mathbf{h}_{1,j},\mathbf{h}_{2,j},\ldots,\mathbf{h}_{N,j}]$ and $\mathbf{C}(t)=[\mathbf{C}_1(t),\mathbf{C}_2(t),\ldots,\mathbf{C}_N(t)]^T$, where $\mathbf{H}_j$ is the frequency selective fading channel matrix for receive antenna $j$, and $\mathbf{C}(t)$ is the coded symbol matrix at time $t$. The superscript $T$ denotes matrix transposition, and the transmitted symbol energy $E_s$ is normalised to one. A guard period of length $L$ is added to avoid inter-frame interference.

The tap gains $h_{i,j}(l)$ are modelled as independent complex Gaussian random variables, with zero mean and variance $\sigma^2_{i,j}(l)$. The total variance for each channel is normalized to one, i.e. $\sum_{l=0}^{L-1}\sigma^2_{i,j}(l)=1$. The channel gains are assumed known perfectly in the receiver.

*B. Performance Bound*

We first derive the pairwise error probability of STTC MLED in channel with arbitrary delay profile for later analysis. Using the MLED receiver, the maximum likelihood decision rule is

$$\hat{\mathbf{c}}=\arg\min_{\mathbf{c}}\sum_{j=1}^{M}\sum_{t=1}^{T}\left|r_j(t)-\mathbf{H}_j\mathbf{C}(t)\right|^2 . \tag{1}$$

Given the channel gains, the probability of erroneously detecting the transmitted code sequence **c** into **c'** given the channel **H** is upper-bounded by

$$P(\mathbf{c}\to\mathbf{c}'\,|\,\mathbf{H})\le\exp\left(-d^2(\mathbf{c},\mathbf{c}')E_s/4N_0\right) \tag{2}$$

with $d^2(\mathbf{c},\mathbf{c}')=\sum_{j=1}^{M}\sum_{t=1}^{T}\left|\mathbf{H}_j\mathbf{E}(t)\right|^2=\sum_{j=1}^{M}\mathbf{H}_j\left[\sum_{t=1}^{T}\mathbf{E}(t)\mathbf{E}^H(t)\right]\mathbf{H}_j^H$

where $\mathbf{E}(t)=[\mathbf{E}_1(t),\mathbf{E}_2(t),\ldots,\mathbf{E}_N(t)]^T$, and $\mathbf{E}_i(t)=[c_i(t)-c_i'(t),\ldots,c_i(t-L+1)-c_i'(t-L+1)]$ is the error vector and the superscript $H$ denotes complex conjugate transposition. Further, let's define $\mathbf{D}_{MLED}$ be the composite error matrix as

$$\mathbf{D}_{MLED} \overset{def}{=} \sum_{t=1}^{T} \mathbf{E}(t)\mathbf{E}^H(t) = \mathbf{Q}^H \mathbf{\Lambda} \mathbf{Q}. \tag{3}$$

Note that the dimension of the matrix $\mathbf{D}_{MLED}$ is $NL \times NL$, hence $\gamma = rank(\mathbf{D}_{MLED}) \leq NL$. The unitary matrix $\mathbf{Q}$ and the diagonal matrix $\Lambda$ (containing the eigenvalues $\{\lambda_1, \ldots \lambda_\gamma\}$ ) are obtained from the singular value decomposition of the non-negative definite Hermitian matrix $\mathbf{D}_{MLED}$. Thus the pairwise error probability can be written as

$$P(\mathbf{c} \rightarrow \mathbf{c}' \mid \mathbf{H}) \leq \prod_{j=1}^{M} \exp\left( -\frac{E_s}{4N_0} \sum_{i=1}^{\gamma} \lambda_i \left\| [\boldsymbol{\beta}_j]_{(i)} \right\|^2 \right)$$

where $[\mathbf{x}]_{(i)}$ denotes the $i$-th row of the matrix $\mathbf{x}$, and $\boldsymbol{\beta}_j = \mathbf{H}_j \mathbf{Q}^H$. This $\boldsymbol{\beta}_j$ has a correlation matrix $E[\boldsymbol{\beta}_j^H \boldsymbol{\beta}_j] = \mathbf{Q} \mathbf{\Omega}_j \mathbf{Q}^H$, where $\mathbf{\Omega}_j = diag(\sigma_{1,j}^2(0), \ldots, \sigma_{1,j}^2(L-1), \ldots, \sigma_{N,j}^2(L-1))$ and $diag(a_i)$ denotes a diagonal matrix with $a_i$ at the $i$-th diagonal entry. Thus the elements of $\boldsymbol{\beta}_j$ possess independent Rayleigh distribution if the channel delay profile is uniform, but is dependent if the profile becomes non-uniform. Now let $\psi$ be the matrix square root of $E[\boldsymbol{\beta}_j^H \boldsymbol{\beta}_j]$ such that $\mathbf{\Psi} = \mathbf{Q}(\mathbf{\Omega}_j)^{1/2} \mathbf{Q}^H$. By using $\psi$ as a whitening matrix, we let $\mathbf{v}_j = \boldsymbol{\beta}_j (\mathbf{\Psi}_j)^{-1}$ such that the correlation matrix for $\mathbf{v}_j$ equals the identity matrix, (i.e. independent and identically Rayleigh distributed). Thus the pairwise error probability can now be written as

$$P(\mathbf{c} \rightarrow \mathbf{c}' \mid \mathbf{H}) \leq \prod_{j=1}^{M} \exp\left( -\frac{E_s}{4N_0} \mathbf{v}_j \mathbf{\Psi}_j^H \mathbf{\Lambda} \mathbf{\Psi}_j \mathbf{v}_j^H \right). \tag{4}$$

By averaging (4) over Rayleigh distribution and using the Hadamard inequality for matrix determinants, the pairwise error probability becomes

$$P(\mathbf{c} \rightarrow \mathbf{c}') \leq \prod_{j=1}^{M} \left( \prod_{i=1}^{\gamma} \left( 1 + \lambda_i \xi_{i,j} \frac{E_s}{4N_0} \right) \right)^{-1}. \tag{5}$$

where $\xi_{i,j} = \left\| [\mathbf{\Psi}_j]_{(i)} \right\|^2 = \sum_{k=1}^{N} \sum_{l=0}^{L-1} \sigma_{k,j}^2(l) \left\| [\mathbf{Q}]_{(i,k*L+l)} \right\|^2$, and $[\mathbf{x}]_{(i,j)}$ denotes the $(i,j)$-th entry of the matrix $\mathbf{x}$. If the channel delay profile from the same transmit antenna to all receive antennas are the same, the $j$ element in the $\xi_{i,j}$ term can be omitted and the probability can then be written as

$$P(\mathbf{c} \rightarrow \mathbf{c}') \leq \left( \prod_{i=1}^{\gamma} \left( 1 + \lambda_i \xi_i \frac{E_s}{4N_0} \right) \right)^{-M} \leq \left( \prod_{i=1}^{\gamma} \lambda_i \xi_i \right)^{-M} \left( \frac{E_s}{4N_0} \right)^{-\gamma M}. \tag{6}$$

Hence the achievable diversity order (the exponent on SNR) is $\gamma M$. Moreover, if the composite error matrix $\mathbf{D}_{MLED}$ is full rank (= $NL$), the maximum achievable diversity order is $NML$. This implies that the MLED receiver for STTC in frequency selective channels attains frequency diversity in addition to spatial and temporal diversity. As the code constraint length controls the composite error matrix $\mathbf{D}_{MLED}$, properly designing the code can make $\mathbf{D}_{MLED}$ full rank and hence achieves the maximum diversity order. Notice that with the total channel power being constrained, the probability in (6) (similarly for (5)) is maximized when $\xi_i = \xi_{i'}, \forall i \neq i'$. This can be obtained if and only if all $\sigma_{k,j}^2(l)$ are the same (i.e. uniform profile channel). Thus in other words, the non-uniformity of the channel delay profile degrades the performance by reducing the coding gain achievable in the STTC system (the first product between $\lambda_i$ and $\xi_i$ of equation (6)).

It is also noted that the above bound is identical for both iterative MAP-ED [2], and the CT-ED [3] schemes. It is because both schemes use the same metric in (1). Hence the signal models for them are identical and the bounds are therefore identical.

## III. STTC OFDM

Instead of using MLED to combat frequency selective fading, another approach is to use OFDM. In this section, we briefly revisit STTC with OFDM and its performance analysis. A more complete study of this work can be found in [6] and interested readers are referred to it.

### *A. Signal Model*

An OFDM system with $K$ subcarriers is applied to the ($N$, $M$) STTC system over frequency selective fading channels discussed in the previous section. Each coded symbol is modulated onto one subcarrier of the OFDM word of its corresponding transmit antenna. Cyclic prefix of length $L$ is added before transmitting the OFDM words simultaneously. The $k$-th subchannel frequency response from transmit antenna $i$ to receive antenna $j$ is

$$H_{i,j}(k) = \sum_{l=0}^{L-1} h_{i,j}(l) \exp(-j2\pi k n_l / K) = \mathbf{h}_{i,j}\mathbf{w}(k) \tag{7}$$

where $\mathbf{w}(k) = [\exp(-j2\pi k n_0 / K), \ldots, \exp(-j2\pi k n_{L-1} / K)]^T$

$$\mathbf{h}_{i,j} = [h_{i,j}(0), h_{i,j}(1), \ldots, h_{i,j}(L-1)] \, ,$$

and $n_l$ is the path arrival time normalized to the OFDM subcarrier spacing, such that $n_l T_f$ is the delay and $1/T_f$ is the OFDM subcarrier spacing.

*B. Performance Bound*

The derivation of the pairwise error probability for uniform profile channel is detailed in [6] and here we restate some key points and generalizes it to the non-uniform profile channel case. The pairwise error probability is the identical to (2), with

$$d^2(\mathbf{c},\mathbf{c}') = \sum_{j=1}^{M}\sum_{k=0}^{K-1}\left|\sum_{i=1}^{N} H_{i,j}(k)E_i(k)\right|^2 = \sum_{j=1}^{M}\mathbf{h}_{i,j}\mathbf{D}_{OFDM}\mathbf{h}_{i,j} \tag{8}$$

where $E_i(k) = c_i(k) - c_i'(k)$ and the composite error matrix $\mathbf{D}_{OFDM}$ now defined as

$$\mathbf{D}_{OFDM} \overset{def}{=} \sum_{k=0}^{K-1}\mathbf{W}(k)\mathbf{E}(k)\mathbf{E}^H(k)\mathbf{W}^H(k) \tag{9}$$

with $\mathbf{E}(k) = [c_1(k) - c_1'(k), \ldots, c_N(k) - c_N'(k)]^T$ and $\mathbf{W}(k)$ is a $NL\times N$ block diagonal DFT matrix with $N$ entries of $\mathbf{w}(k)$. Using similar derivation in the above section, the pairwise error probability for STTC OFDM in frequency selective Rayleigh fading channel is identical to (6), where $\gamma$ and $\lambda_i$ are now the rank and eigenvalues of $\mathbf{D}_{OFDM}$ respectively. Similarly, the $\mathbf{Q}$ matrix in $\xi_i$ is the unitary matrix obtained from the singular value decomposition on $\mathbf{D}_{OFDM}$. Since the dimension of $\mathbf{D}_{OFDM}$ is $NL\times NL$, its maximum rank will be $NL$. Thus, the maximum achievable diversity order in STTC OFDM in frequency selective fading channel is $NML$.

## IV. SPACE-TIME TRELLIS CODE DESIGN

In this section, the code design issues for STTC with MLED and OFDM in frequency selective fading channels are discussed. First, let's define the effective length of a code as $d_{eff}$, which is the minimum number of incorrect trellis transitions due to one bit error. This parameter is controlled by the code constraint length. In particular, to achieve a certain $d_{eff}$, the code constraint length must be at least $d_{eff}$ -1. Hence for a STTC with rate $r$ bit/s/Hz, the number of coding states required to achieve a certain $d_{eff}$ is $2^{r(d_{eff}-1)}$. We further let $\mathbf{E}$ be the error matrix as $\mathbf{E} = [\mathbf{E}(1), \mathbf{E}(2), \ldots, \mathbf{E}(T)]$ ($K$ instead of $T$ for OFDM), and define $p$ as the minimum column rank of $\mathbf{E}$ due to one bit error. In other words, $p$ is the minimum number of linearly independent non-zero error vector $\mathbf{E}(t)$ due to one bit error. This $p$ is very useful in code design analysis, as it equals to the rank of $\mathbf{D}_{MLED}$. It also equals to $\mathbf{D}_{OFDM}$ as each of the DFT matrix $\mathbf{W}$(k) is full rank and independent to others at different values of $k$. Since the rank of these composite error matrices equals the diversity order of the system per receive antenna, this $p$ effectively controls the performance.

*A. STTC MLED*

For STTC in frequency flat fading channel ($L$=1), $p$ equals the code effective length $d_{eff}$. Therefore, the code design criteria is to make $p$ at least $N$ and the error vector $\mathbf{E}(t)$ be linearly independent for different values of $t$ to ensure that $\mathbf{D}_{MLED}$ is full rank (= $N$).

However for time domain trellis equalization in frequency selective fading channel, one error symbol corresponds to an incorrect trellis path with an additional $L$-1 incorrect trellis transitions. This can be viewed as if the erroneous symbol propagates through the channel tap delay line and create $L$-1 more error instances. Thus for STTC without interleaving, the number of non-zero error instance $p$ is increased by $L$-1, i.e. $p = d_{eff} + L - 1$. Therefore the maximum rank of $\mathbf{D}_{MLED}$ equals the minimum of ($d_{eff} + L - 1, NL$). Hence, to achieve the maximum diversity order, the code must have $d_{eff} \geq NL - L + 1$ and all $\mathbf{E}(t)$ due to one symbol error must be linearly independent.

For system with interleaving, the same interleaver structure should be used for all transmit antennas to preserve the rank property of the code. Then the consecutive error symbols due to one bit error are distributed over the frame. When the interleaver distance is larger than $L$, each error symbol will have $L$-1 more error instances due to the channel. Thus, the total number of non-zero $\mathbf{E}(t)$ instances is $p = d_{eff} L$, and the matrix $\mathbf{D}_{MLED}$ will have a maximum rank of $\min(d_{eff} L, NL)$. Therefore one can achieve the maximum diversity order with a weaker code when interleaving is applied.

It must be noted that the decoding complexity for system with interleaving (MAP-ED) is much higher than the one without interleaving (CT-ED). The MAP equalizer trellis consists of $2^{rN(L-1)}$ states and $2^{rN}$ transitions regardless of the code used. In addition, the MAP decoder trellis has $2^{r(d_{eff}-1)}$ states and $2^r$ transitions. However the CT-ED will only have $2^{r(d_{eff}+L-2)}$ states and $2^r$ transitions. Moreover, MAP detectors require more computations for its metric than ML detectors. Hence in achieving a better performance by interleaving, the receiver complexity will be increased.

*B. STTC OFDM*

For OFDM, the frequency selective fading channel is converted to a number of parallel flat fading subchannels. Therefore, the minimum number of non-zero error vector $p$ equals the effective length of the code $d_{eff}$. Hence in order to achieve the maximum diversity order, the code must have an effective length $d_{eff} \geq NL$.

Now consider the effect of interleaving in STTC OFDM system. With the block diagonal structure of the DFT matrix $\mathbf{W}$(k), the $\mathbf{D}_{OFDM}$ matrix can be rewritten as a Kronecker product

$$\mathbf{D}_{OFDM} = \sum_{k=0}^{K-1} \left[\mathbf{E}(k)\mathbf{E}^H(k)\right] \otimes \left[\mathbf{W}(k)\mathbf{W}^H(k)\right] \overset{def}{=} \sum_{k=0}^{K-1} \breve{\mathbf{E}}(k) \otimes \breve{\mathbf{W}}(k).$$

When no interleaving is applied, the error instances $k$ will be consecutive. If the signal paths are clustered in a short period of time, the difference between paths arrival time $n_l$ is small comparing to the DFT size $K$. Thus the $\breve{\mathbf{W}}(k)$ matrices at consecutive $k$ will be very similar to each other and can be approximated to be the identical. Hence,

$$\mathbf{D}_{OFDM} \cong \left[\sum_{k=0}^{K-1} \breve{\mathbf{E}}(k)\right] \otimes \breve{\mathbf{W}}(k).$$

As the rank of $\breve{\mathbf{W}}(k)$ is one, the rank of this $\mathbf{D}_{OFDM}$ matrix will reduce to $N$ and thus reducing the diversity order of the system.

By interleaving the symbols before transmission, consecutive symbols will not be modulated onto consecutive subchannels. Hence the consecutive error symbols due to one error bit are distributed over non-adjacent subchannels, i.e. the subchannel number $k$ for these error instances are separated farther apart. Thus, the $\breve{\mathbf{W}}(k)$ matrices for these error instances will be more different with each other and the rank of the $\mathbf{D}_{OFDM}$ matrix can be maintained. Although the clustering of path arrivals is uncontrollable, interleaving will avoid the problem of rank deficiency that leads to a poorer performance.

## V. COMPARISONS

In this section, we compare and discuss the STTC MLED and OFDM schemes in the perspective of performance, receiver complexity and robustness. Several findings are addressed and each finding is verified through Monte Carlo simulation. We used a (2,1) QPSK STTC system for simulation and the channels have symbol-spaced delay with two independently Rayleigh faded taps ($L$=2). For all simulations except those in part F, the channels delay profile is assumed uniform. Hence, the maximum achievable diversity order for this system is 4. In order to have a fair comparison, we consider the same channel with the same frame length for both schemes with 128 symbols per transmit antenna (MLED: $T$=128; OFDM: $K$=128). Notice that the signal-to-noise ratio for OFDM does not include the power used for cyclic prefix. For simplicity, we denote a system with $s$ number of coding states by $s$-st. Moreover as all discussions in this section are focused on STTC, we simply use the term MLED and OFDM to represent the MLED and OFDM schemes respectively.

### *A. Maximum Diversity Order*

As shown in the previous section, both the MLED and OFDM schemes will have the same maximum achievable diversity order $NML$ over the same channel. It means that both schemes can obtain all spatial, temporal and frequency diversity. To verify this finding, the STTC systems with different codes and detection schemes that should achieve the maximum diversity order are simulated and their frame error rates are plotted in Figure 1. First we show the system performances for 4-st MAP-ED ($d_{eff}$ = 2) with random interleaving (w/i) at third iteration,16-st CT-ED ($d_{eff}$ = 3), and 64-st OFDM ($d_{eff}$ = 4) w/i. All these schemes achieve the maximum diversity order (Δ=4) and hence should have a similar slope in the frame error rate plot at high signal-to-noise region. From the slopes of the plots, this finding is verified. The performance of 4-st STTC over flat fading channel (Δ=2) is also plotted for reference. As its frame error rate plot has a much shallower slope, it supports the argument on achieving frequency diversity in the frequency selective fading channels.

*B. Maximum Diversity Order Achieving Code*

Also from the previous section, in order to achieve the maximum diversity order, the MLED with and without interleaving requires a code with $d_{eff} \geq N$ and $d_{eff} \geq (NL - L + 1)$ respectively, but the OFDM requires one with $d_{eff} \geq NL$. Since $L \geq 1$, the OFDM scheme will require a stronger code than MLED, where interleaving in MLED reduces the coding state requirement. From the previous simulation results, as the slope of the frame error rate plot for 4-st MAP-ED w/i, 16-st CT-ED, and the 64-st OFDM w/i are similar, they have the same diversity order. Therefore, it supports this finding that a weaker code is required for MLED than OFDM, where the MLED with interleaving (MAP-ED) requires less than the one without (CT-ED).

*C. Decoding Trellis Complexity*

To achieve a certain diversity order Δ, the code effective lengths and the number of decoding trellis states required to achieve full diversity for the MLED scheme without interleaving and the OFDM scheme are listed in Table 1. The decoding trellis complexities for MLED CT-ED and OFDM are both $2^{r(\Delta-1)}$. Thus to achieve a certain diversity order, the decoding trellis complexities are identical for both MLED CT-ED and OFDM. As shown in the previous set of results, both 16-st CT-ED and 64-st OFDM w/i achieves the same diversity order, and both require the same decoding trellis complexity with 64 states and 4 branches.

*D. Achievable Diversity Order of a Code*

Consider the code has an effective length $d_{eff}$ that is smaller than the maximum diversity order. The diversity order achieved by this code in MLED with and without interleaving, and in OFDM are $d_{eff}L$, $d_{eff}+L$-1, and $d_{eff}$, respectively. Hence, since $L \geq 1$, MLED achieves higher diversity order than OFDM when the same non-maximum diversity order attainable code is used. To verify this, the STTC systems with the same 16-st code are simulated for MLED and OFDM with interleaving, where they should obtain a different diversity order of 4 and 3 respectively. Their frame error rates are plotted as solid lines in Figure 2 and it can be seen that the CT-ED outperforms OFDM w/i. From the slope of the frame error rate, it showed that the MLED achieve a higher diversity order than OFDM w/i, and supported our finding.

In addition, the performances of these detectors with the same code in 3-tap channels are plotted in dashed lines. Notice that the performance of OFDM is very similar to the 2-tap channel case, and the performance of MLED in 3-tap channel has significant improvement. This is because now the MLED achieves a diversity order of 5, instead of 4 in the 2-tap case. Hence, this further supports that unlike MLED, the performance of OFDM does not improve significantly when the channel tap length is increased. This argument becomes important when the number of channel taps is large, as the maximum diversity order will be so high that no practical code can achieve the required effective length. In such cases, MLED will achieve a much higher diversity order than OFDM with the same code. However, although MLED achieves a higher diversity order than OFDM, its decoding complexity is also higher than OFDM and increases exponentially with the channel tap length.

*E. Interleaving*

As discussed in the previous sections, the use of interleaver in MLED increases the diversity order for codes that cannot attain the maximum diversity order. However, for OFDM, the interleaver is used to avoid rank deficiency. The interleaver distance for MLED should only be larger than $L$ but must be as large as possible for OFDM. Therefore, it suggests a mandatory use of interleaver in OFDM and a more ideal interleaver design is required. To show the importance of interleaving in OFDM, we simulated the OFDM without interleaving (wo/i) and plotted the result also on Figure 2. Notice that the performance of OFDM without interleaving do have a similar slope with STTC in flat fading case, where both are having a smaller slope than OFDM with interleaving. Hence it suggests a smaller diversity order and supports our arguments on rank deficiency in case of no interleaving for OFDM. Moreover, OFDM without interleaving performs worse than the same code in flat fading channel with ML trellis detection. This is because for no interleaving, the correlation between the DFT matrices at the error instances is increased, hence reducing the eigenvalues of $\mathbf{D}_{OFDM}$ in (9). Thus, the coding gain is reduced which results in a poorer performance. This further showed the importance of interleaving for OFDM.

*F. Performance in Non-Uniform Profile Channel*

From our derivations of the pairwise error probability, the performance of both MLED and OFDM will be degraded if the channel has non-uniform profile. To verify this, simulations are performed for two-tap channels with average power 0.8 in the first tap and 0.2 in the second tap. The code used for MLED and OFDM are 16-st and 64-st respectively, such that both schemes achieve the maximum diversity. The frame error rates for the uniform and non-uniform (NU) profile channels are plotted as solid and dashed line in Figure 3. Simulation results show that both MLED and OFDM with interleaving suffer similar performance degradation due to the channel non-uniformity.

On the other hand, the OFDM without interleaving performs slightly better in non-uniform profile channel than the uniform one. This can be explained using the bound derived in previous section. For non-interleaved OFDM system, rank deficiency occurs in the $\mathbf{D}_{OFDM}$ matrix even for a full diversity order achieving code. Thus, only a few eigenvalues of the $\mathbf{D}_{OFDM}$ matrix are non-zero. If the $\xi_i$ for those non-zero eigenvalues are increased, the first product (coding gain) in (6) will also be increased causing an improvement in performance. As some of the $\xi_i$ in non-uniform profile channels are larger than those in the uniform ones, the coding gain product in the pairwise error probability (6) will be larger than the uniform channel cases, which leads to a better performance.

*G. Robustness and Implementation Issues*

It is known that the major advantage of OFDM over MLED schemes is its tap-length independent complexity and hence robust to different environment. This advantage is more important in long tap channels, where MLED requires a much higher complexity. However, the above findings suggest that to achieve the same diversity order (i.e. similar performance), the OFDM scheme will not have any advantage in complexity over MLED. In addition, it must be noted that OFDM requires extra complexity in the DFT (or FFT) operations, channel estimation for each subcarrier, and extra power usage in cyclic prefix insertion. Moreover, the OFDM scheme uses multicarrier transmission and hence has the problem of high peak to average ratio and sensitivity to frequency offset and phase errors. Furthermore, as OFDM requires a higher state code than MLED, designing a code to maintain the desired diversity order is more difficult. Together with the required ideal interleaver design, the OFDM scheme is not preferable for short tap channel, especially comparing to STTC MLED system with channel order estimation [7]. On the contrary, for long tap channel where the full exploration of diversity order is not expected, OFDM is more feasible. It is because the receiver complexity of OFDM scheme only depends on the code constraint length, and is independent to the channel tap length. In addition, if the channel is known at the transmitter, adaptive modulation can be used to improve the performance for OFDM [8].

## VI. Conclusions

In this paper, the performances and code design issues between the STTC MLED and OFDM schemes are compared both analytically and empirically. It was found that the achievable diversity order depends on the effective length of the applied code. Moreover, the maximum achievable diversity orders of these two schemes are both $N \times M \times L$, but the STTC MLED requires a weaker and less complex code than the STTC OFDM to achieve it. Also, the STTC MLED can achieve a higher diversity order than the STTC OFDM when the same non-maximum diversity order achievable code is applied. In order to achieve a certain amount of diversity order, different codes will be required for the two schemes but will have the same decoding trellis complexity. Interleaving is crucial for STTC OFDM to avoid diversity order reduction due to path clustering, but to increase the diversity order achievable by a code in STTC MLED. Simulation results are presented and support our arguments. These arguments suggest a preference of STTC MLED over STTC OFDM in short tap channel environment, and OFDM over MLED in long tap channels. In addition to these comparisons, we found that the non-uniformity of the channel profile will degrade the system performance for both schemes.

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| | MLED | OFDM |
|---|---|---|
| Effective Length | $d_{eff} \geq (\Delta - L + 1)$ | $d_{eff} \geq \Delta$ |
| Decoder Trellis States | $2^{r(d_{eff}+L-2)}$ | $2^{r(d_{eff}-1)}$ |

Table 1: Code effective length and number of decoding trellis states for the two schemes.

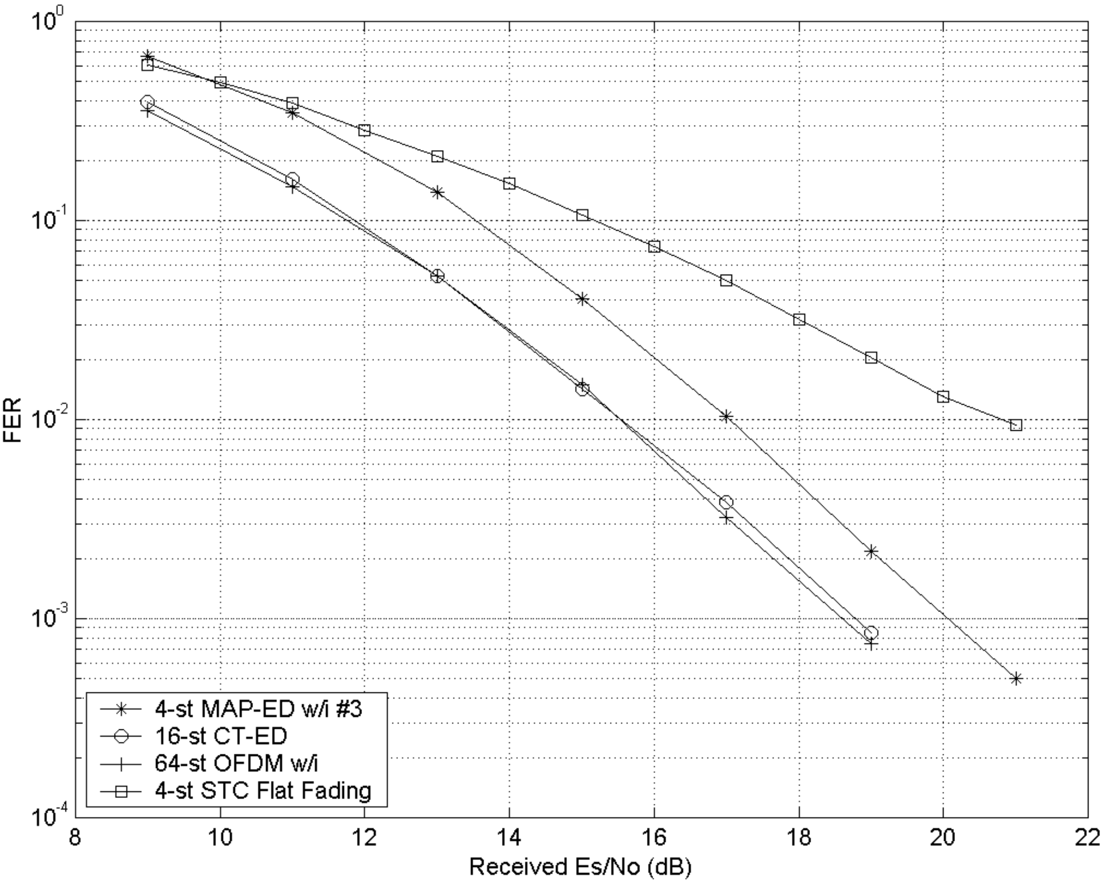


Figure 1: (2,1) space-time code with different approaches over frequency selective channels to verify the maximum achievable diversity order.

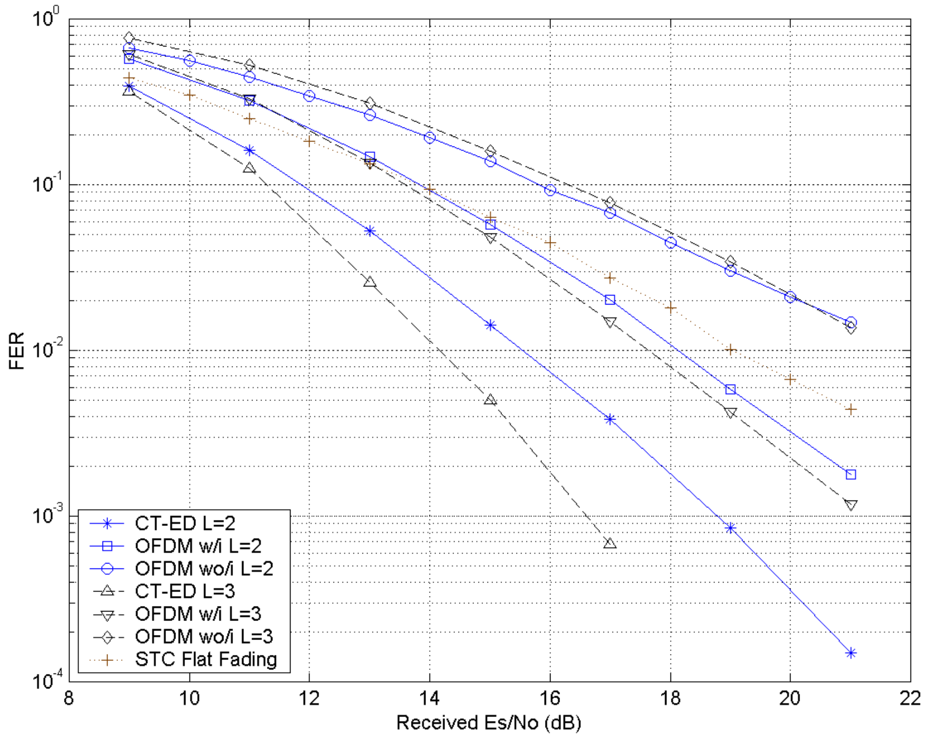


Figure 2: (2,1) space-time code over 2-tap and 3 tap channels with 16 states code for MLED (CT-ED) and OFDM with and without interleaving.

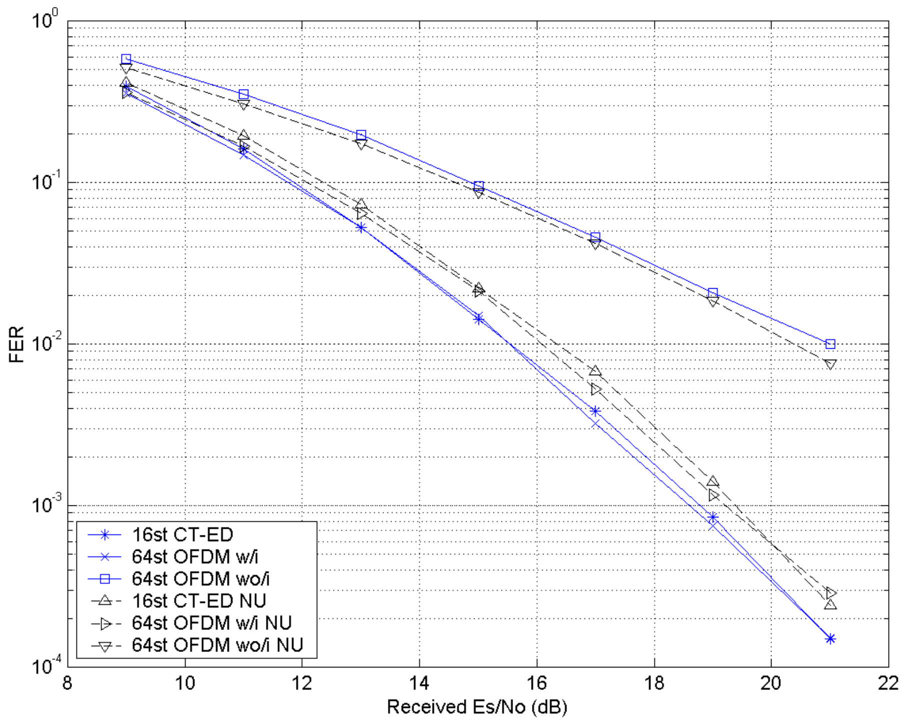


Figure 3: Performance comparison of (2,1) space-time code over 2-tap channels in uniform and non-uniform profile channels.