Analysis and Design of Financial Portfolios using Noise Clipping Algorithms (MM1b-13)

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INTRODUCTION

● Overview

Over the last decade, there has been an explosion in the amount of data available in the stock market. Finding how the stocks within a financial portfolio are associated with each other can lead to profitable stock trading strategies. Traditionally, sample covariance is used to measure the associations among data. However, the accuracy of sample covariance is greatly affected by factors like the sample size and the noise interference in data. Thus, some more robust estimation methods are needed to solve the problem. In this particular project, a more advanced estimation algorithm called noise clipping is implemented to solve the problem. This FYP is to apply noise-clipping algorithm into the estimation of associations among different stocks within a financial portfolio with the hope of optimizing the portfolio.

● Aims and Objectives

The aim of this project is to develop a more accurate estimate of covariance matrix by adopting noise clipping algorithm to assist achieve a better portfolio optimization under the Markowitz Model. By adopting the noise-clipping algorithm, more robust estimation can be made in finding the hidden associations among data. By achieving a better portfolio optimization, values can be added into existing research area for the benefit of investors.

METHODOLOGY

● Project Plan by Stages

➢ Stage 1: Data Generation and Analysis
➢ Stage 2: Limitations of Sample Covariance and Observation Window Size Determination
➢ Stage 3: Effectiveness Testing for Noise Clipping Algorithm

We will mainly focus on the two main results in Stage 2.

● Evaluation Metric

➢ For the framework of global maximum variance portfolio given by Markowitz, the performance is measured by the risk of such optimized portfolio.

➢ The formulation is given below:

\[
\rho_{\text{opt}}(\mathbf{W}) = \mathbf{w}^T \mathbf{C}_W \mathbf{w}, \quad \mathbf{w}^T \mathbf{1} = 1
\]

where \(\mathbf{C}_W\) is the covariance matrix of the assets.

Mathematical Formulation:

where \(\mathbf{W}\) is the estimation used to estimate the true covariance matrix. In our discussion, the estimation used for comparison are the sample covariance matrix \(\mathbf{C}\) and the covariance matrix adopting noise clipping algorithm \(\mathbf{C}_{\text{clipped}}\). The above two estimation will be defined below.

➢ Sample covariance matrix \(\mathbf{C}\):

\[
\mathbf{C} = \frac{1}{n-1} \left( \sum_{i=1}^{n} \mathbf{R}_i \mathbf{R}_i^T - \mathbf{R}_n \mathbf{R}_n^T \right)
\]

where \(\mathbf{R}_i\) is the N-by-M asset returns matrix, \(N\) is the number of assets, \(M\) is the observation window length.

➢ Covariance matrix adopting noise clipping algorithm \(\mathbf{C}_{\text{clipped}}\):

\[
\mathbf{C}_{\text{clipped}} = \mathbf{C} - \sum_{i=1}^{n} \Delta_i \mathbf{R}_i \mathbf{R}_i^T - \frac{1}{n} \sum_{i=1}^{n} \mathbf{R}_i \mathbf{R}_i^T \mathbf{R}_n \mathbf{R}_n^T
\]

where \(\Delta_i\) is the number of eigenvectors satisfying \(\lambda_i > \lambda_{\text{threshold}}\) and \(\lambda_{\text{threshold}}\) is the number of the sample covariance matrix.

RESULTS

● Simulation Data

From the above figure showing the results of both for all the three models, we can see that the performance of the noise clipping algorithm, in general, is better than that of the sample covariance measure, especially when the observation window size is relatively small. Remarkable improvement in portfolio risk can be achieved from the above figures.

● Real Data (Daily Returns of Hang Seng Index)

From the above figure showing the result for real data, we can see that the noise clipping algorithm is effective in improving the portfolio optimization. The portfolio risk decreases as we adopt this algorithm.

● Brief Conclusion

The performance improvements of noise-clipping algorithms are prominent above from the tests result illustrated in previous section. For all the three models and the real daily returns tested, the portfolio risk decreases when this algorithm is adopted to estimating the covariance matrix. Also, what is remarkable is that unlike the sample covariance measure, this new method has low sensitivity to the size of observation window, which has provided a good solution for eliminating the limitations brought by sample covariance.