DESIGN AND PERFORMANCE ANALYSIS
OF MIMO/OFDM SYSTEMS
WITH LIMITED CSI FEEDBACK

PROJECT CODE: VL3a-06

Student: TIN Yuk Shing
Supervisor: Prof. Vincent Lau
OVERVIEW

Multiple Input Multiple Output (MIMO) wireless communication systems will widely be adopted in the next generation. The knowledge of channel state information (CSI) at the transmitter offers tremendous advantage for MIMO systems. However, in practice, only limited CSI can be acquired to generate CSI feedback to the transmitter (CSIT). Therefore, this project was to design how to effectively utilize the limited CSI to generate a few CSI feedback bits and generate adaptation schemes for the systems.

The project aimed to determine an efficient algorithm for quantization of the obtained CSI in the receiver (CSIR) to assign different combinations of feedback bits. Based on different feedback bits received, corresponding adaptations were determined.

SYSTEM MODEL AND DESIGNS

MIMO Channel Model \((n_T\) transmitter antennas, \(n_R\) receiver antennas\)

\[
Y = HX + Z
\]

- \(Y\) - received symbols in \(n_R \times 1\) vector;
- \(H\) - instantaneous CSI in \(n_R \times n_T\) complex Gaussian random variables;
- \(X\) - transmitted signal \(n_T \times 1\) vector;
- \(Z\) - noise signal in \(n_R \times 1\) vector.

Frequency Interleaver Design

In figure 2, if the first transmission was failed, the transmitted symbols \{A, B, C, D\} were reordered in the 2\(^{nd}\) transmission and the following retransmissions if necessary. Suppose the subcarrier in blue was very poor and the symbol A was the decoding bottleneck in the 1\(^{st}\) transmission. However symbol A was put on another subcarrier in red, in the next transmission, to increase the chance of successful decoding.
The interleaving patterns were chosen to maximize the mutual information.

$$\max_{m=1}^M \sum_{i=1}^K \log_2 \left( 1 + \frac{|h_i^m|^2 + \sum_{j=1}^{n-1} |h_{i,j}|^2}{\sigma_z^2} p \right)$$

where $h_{i,1}, h_{i,2}, \ldots, h_{i,(n-1)}$ - channel gains of the i-th transmitted symbol in the first n-1 times transmissions

$h_i^m$ - channel gain of the i-th transmitted symbol when the m-th interleaving pattern is used

$K$ - total number of subcarriers; $p$ - transmission power; $\sigma_z^2$ - power of noise

**PROBLEM FORMULATION**

The optimality of the frequency interleaver patterns consisted of the following aspects:

1. Optimization of the CSIR partitions
2. Minimization of the PER of the Systems

**SOLUTION**

**Rearrangement of the symbols**

{eg, 5 subcarriers to send 5 symbols (A B C D E)}

Initially, (A B C D E) were arranged in normal order. If the received codeword was wrong, symbols were rearranged by swapping from the weakest channel to the strongest one, second weakest to second strongest, ... and re-sent. New received symbols were optimally combined with old symbols received based on their channel gains. For example, symbol E in subcarrier 5 (weakest) was most likely to cause errors and would be swapped into subcarrier 3 (strongest). If the received codeword was wrong again, symbols were re-arranged again but considering cumulative channel gains of the same symbols until it was correctly decoded.

Then, the received symbol $r_i$ was optimally combined:

$$r_i = h_{i,1}r_{i,1} + h_{i,2}r_{i,2} + \ldots + h_{i(n-1)}r_{i(n-1)} + h_{i,m}r_{i,m}$$

where $h_1, h_2, \ldots, h_{(n-1)}$ - channel gain of the i-th symbol; $h_{im}$ - channel gain of the current symbol.
Quantization of the CSIR with Limited Feedback Bits

Lloyd’s algorithm was adopted and divided into 2 steps:

Define distortion function

\[
d(h,i) = \sum_{k=1}^{K} \left\| \hat{H}_{ik} - h_k \right\|^2
\]

**Step 1:** Determine the optimal policy \( \{p_1, p_2, ..., p_{2^{C_{fb}}} \} \)

\[p_i = \arg \min_{p_i} \{ \epsilon [d(h,i)] \} \quad \text{where} \quad h \in h_n\]

**Step 2:** Determine the optimal partition \( \{\hat{H}_i\} \) from the policy \( \{p_i\} \).

\[\hat{H}_i = \{h: d(h,i) \leq d(h,j); \text{for all } j, i \in [1, ..., 2^{C_{fb}}], i \neq j\}\]

(Lloyd’s algorithm cannot be guaranteed to converge to the global optimum)

**Key Simulation Results**

**Adaptive frequency interleaver**

Interleaving patterns were generated randomly and the choices of pattern were determined by the maximum value of the summation of mutual information of each symbol for each pattern.

**Adaptive frequency interleaver with limited CSIT feedback**

Same as adaptive frequency interleaver but the patterns were generated using Lloyd’s algorithm.

**Adaptive frequency interleaver with perfect CSIT feedback**

Assume the complete knowledge of subcarrier gains at the transmitters.

![Figure 4 BER vs SNR](image1.png)

![Figure 5 PER vs SNR](image2.png)

Figure 4 & 5 showed the adaptive frequency interleaver with limited CSIT outweighed the adaptive frequency interleaver when the SNR was above certain threshold.

\( \hat{H}_{ik} \) - \( k \)-th subcarrier of \( i \)-th centroid of CSIR for \( i=1,2,...,2^{C_{fb}} \)

\( h_k \) - \( k \)-th subcarrier of a channel in CSIR for \( k=1,2,...,K \)

\( C_{fb} \) - feedback capacity

= number of feedback bits

\( K \) - number of subcarrier of a channel

\( \hat{H}_i \) was initially chosen randomly.